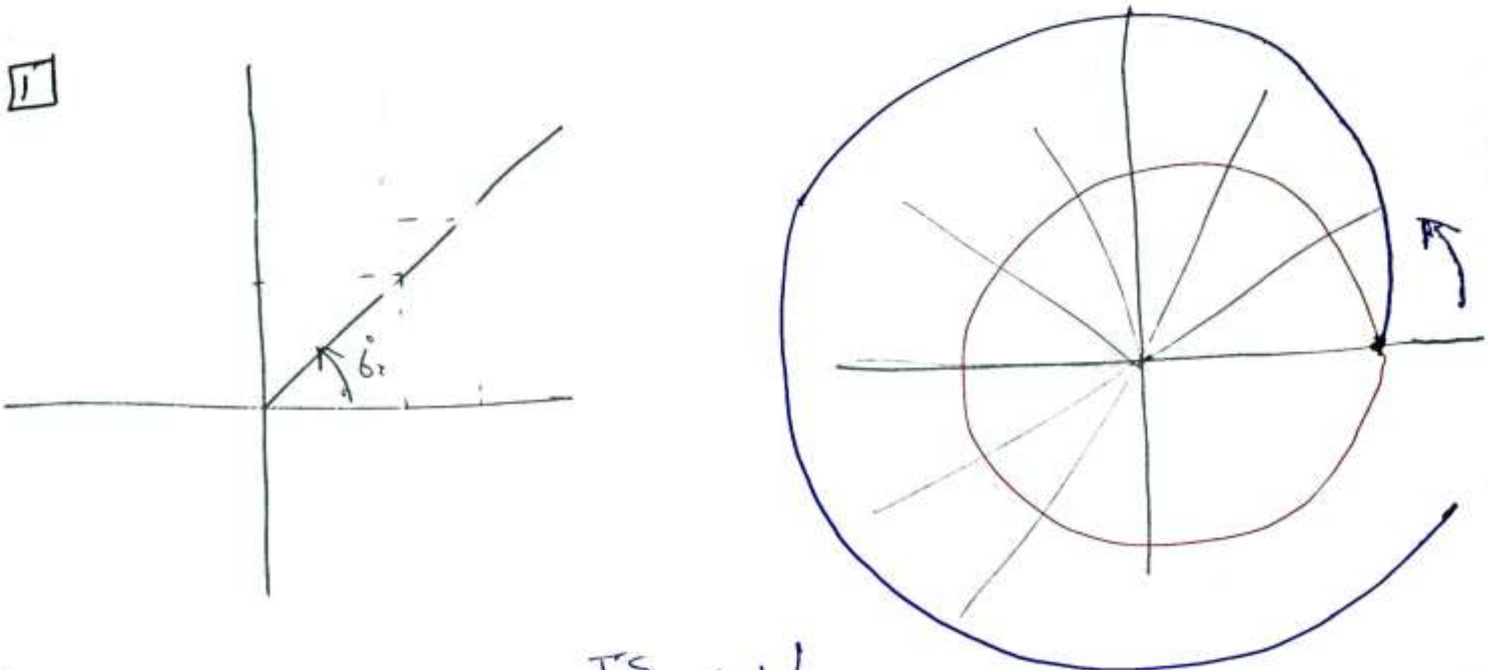
Lec 4 Digital Control

* Map the following lines to Z-Plane

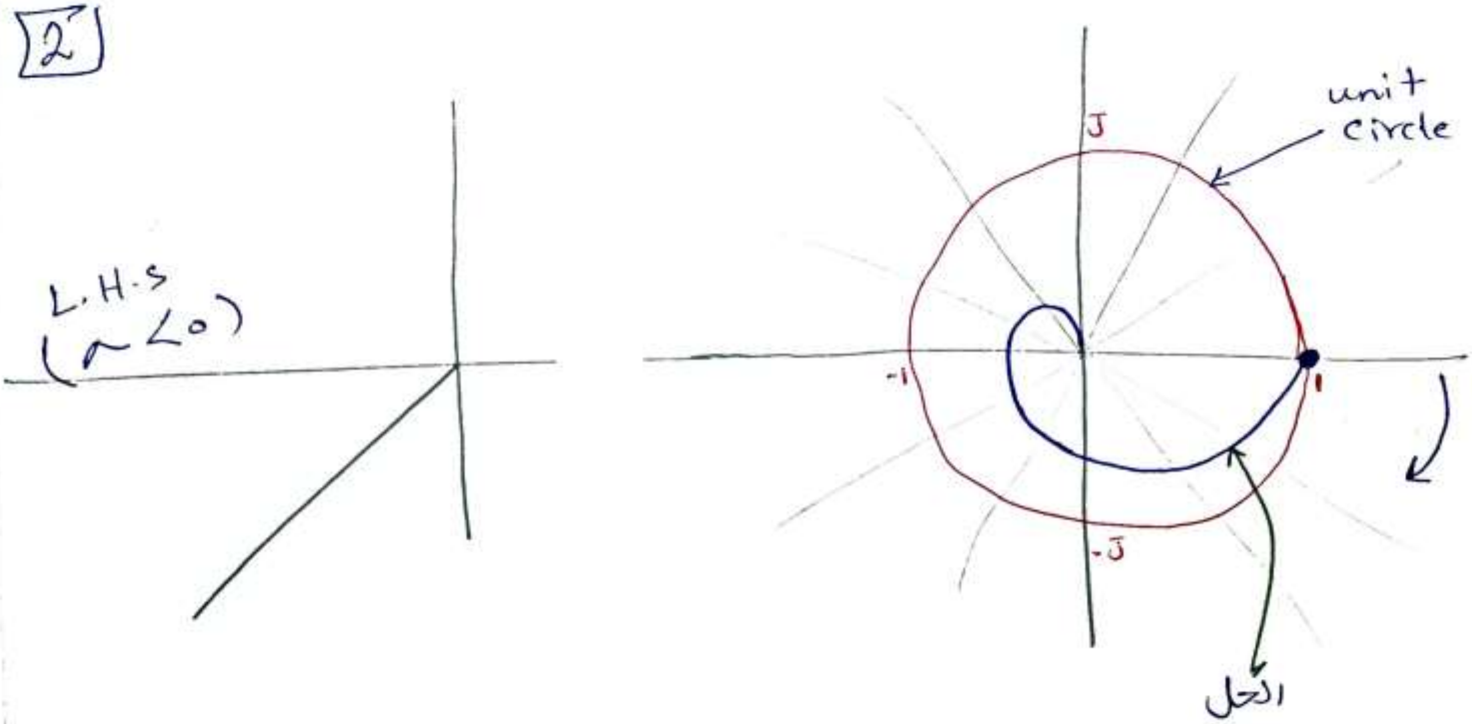
1

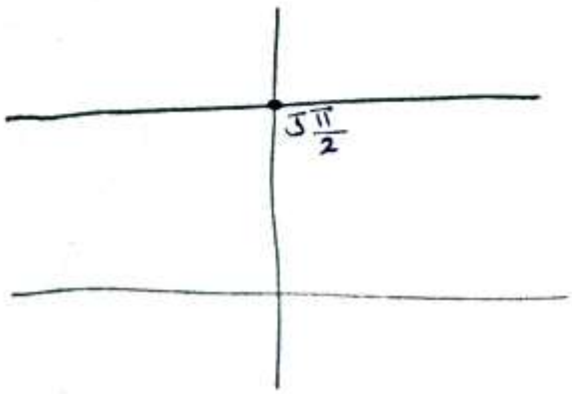


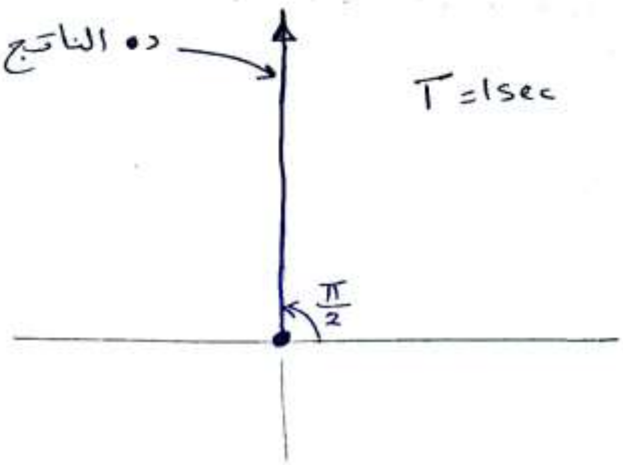
$s = \sigma + j\omega \Rightarrow Z = e^{Ts} = 1 \angle$

$= r \angle \theta$

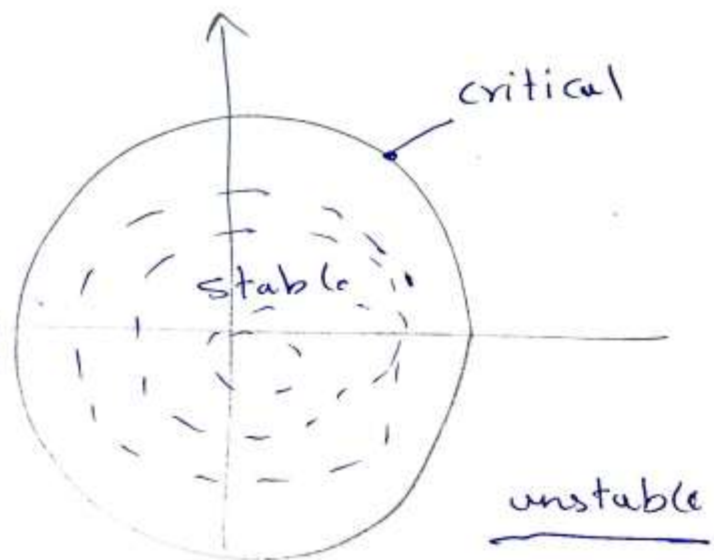
2







$r = |z| =$
- $r = 1$ (critical)
- $r < 1$ (stable Pole
- $r > 1$ (unstable Pole)



check system stability

[I] Poles locations

Ex:1 Ch. equation

$$z^2 - z + 0.5 = 0$$

Poles $\Rightarrow Z_{1,2} = 0.5 \pm J0.5$

$$|Z_{1,2}| = \sqrt{(0.5)^2 + (0.5)^2} = 0.707 < 1 \quad \text{(stable)}$$

Ex2 ch. equation

$$(z-0.5)(z+0.7)(z-1)=0$$

Poles $z_{1,2,3} = 0.5, -0.7, 1$

$$|z| = \underset{<1}{0.5},\ \underset{<1}{0.7},\ \underset{=1}{1} \Rightarrow \text{critically stable}$$

← طالما مفيش حاجة خارجة دائرة الوحدة.

Ex:3

$$(z-0.5)(z+0.7)(z-1)(z+1.5)=0$$

Poles $z = 0.5, -0.7, 1, -1.5$

$$|z| = \underbrace{\underset{<1}{0.5},\ \underset{<1}{0.7}}_{\text{(stable poles)}},\ \underset{\text{(critical)}}{\underset{=1}{1}},\ \underset{\text{(unstable)}}{\underset{>1}{1.5}} \Rightarrow \text{system unstable.}$$

$\boxed{2}$ using bilinear transformation:-

$$z = e^{Ts} = e^{\frac{T}{2}s} \cdot e^{\frac{T}{2}s} = \frac{e^{\frac{T}{2}s}}{e^{-\frac{T}{2}s}}$$

→ Taylor Expansion

$$e^{x} = 1 + x + \frac{x^2}{2!} + \frac{x^3}{3!} + \cdots$$

for small values of x

$$e^{x} \simeq 1 + x$$

$$z = \frac{e^{\frac{T}{2}s}}{e^{-\frac{T}{2}s}} = \frac{1 + \frac{T}{2}s}{1 - \frac{T}{2}s}$$

assume $\frac{T}{2}s = r$

$$\boxed{z = \frac{1+r}{1-r}}$$

ـ كأنّي بعمل (mapping)
(z-domain) ل (r-domain)

→ stability can be checked using Routh method.

Ex ch. equation

$$z^3 + 3.3\,z^2 + 3z + 0.8 = 0$$

→ check stability using bilinear Transformation

$$z = \frac{1+r}{1-r}$$

$$\left(\frac{1+r}{1-r}\right)^3 + 3.3\left(\frac{1+r}{1-r}\right)^2 + 3\left(\frac{1+r}{1-r}\right) + 0.8 = 0$$

بالضرب في $(1-r)^3$

$$(1+r)^3 + 3.3(1+r)^2(1-r) + 3(1+r)(1-r)^2 +$$
$$0.8(1-r)^3 = 0$$

$$(1+r)(1+2r+r^2) + 3.3(1-r)(1+2r+r^2) +$$
$$3(1+r)(1-2r+r^2) + 0.8(1-r)(1-2r+r^2) = 0$$

$$\boxed{r^3 + 9r^2 - 9r - 81 = 0} \quad \left(\begin{array}{c}\text{ch.equation in} \\ r\text{-domain}\end{array}\right)$$

r^3	1	-9
r^2	9	-81
r^1	0	
r^1	18	
r^0	-81	

A(r)

$$A(r) = 9r^2 - 81$$

$$\frac{dA(r)}{dr} = 18r$$

(system unstable)

ـــ الإشارة لم تتغير في آخر خطوة من +ve لـ (-ve)

$\boxed{\text{Ex2}}$ $z^3 - 0.2z^2 - 0.25z + 0.05 = 0$

$$z = \frac{1+r}{1-r}$$

<u>ch. eqn.</u>

$$\left(\frac{1+r}{1-r}\right)^3 - 0.2\left(\frac{1+r}{1-r}\right)^2 - 0.25\left(\frac{1+r}{1-r}\right) + 0.05 = 0$$

$$(1+r)^3 - 0.2(1+r)^2(1-r) - 0.25(1+r)(1-r)^2$$
$$+ 0.05(1-r)^3 = 0$$

$$\boxed{0.9r^3 + 3.6r^2 + 2.9r + 0.6 = 0}$$ → ch. equation in r-domain

r^3	0.9	2.9
r^2	3.6	0.6
r^1	2.75	
r^0	0.6	

* There are no signal changes in first column: system stable.

3 using Jury Test

مع (Jury) على مصفوفة قدر يحسب بيها ال (stability)
← شغالين ال (z-domain).

assume That the ch.eqn. ⇒ $F(z)$

$$F(z) = 1 + \overline{GH}(z) = 0$$

→ General form

$$\boxed{\begin{array}{l} F(z) = a_n z^n + a_{n-1} z^{n-1} + a_{n-2} z^{n-2} + \cdots + a_0 \\ \quad \hookrightarrow F(z) = 1 + \overline{GH}(z) \end{array}}$$

→ The system to be stable

1) $F(1) > 0$

2) $(-1)^n F(-1) > 0$ → n → system order.

3) $|a_0| < |a_n|$

← الثلاثة نقط دي تستخدم في حالة (2nd order system)

لكن لو (n → system order) أكبر من 2 نستخدم Jury-matrix
و ليها شروط إضافية.

Construct Jury array

row no.	z^0	z^1	z^2			z^{n-1}	z^n
1	a_0	a_1	a_2	$\cdots$		a_{n-1}	a_n
2	a_n	a_{n-1}	a_{n-2}	$\cdots$		a_1	a_0
3	b_0	b_1		$\cdots$		b_{n-1}	
4	b_{n-1}	b_{n-2}		$\cdots$	b_1	b_0	
$2n-3$	$\boxed{r_0}$	$\boxed{r_1}$	$\boxed{r_2}$				

لاحظ إن عدد العناصر قد أصبح أقل بمقدار 1

(ثلاثة عناصر) آخر صف

$$b_0 = \begin{vmatrix} a_0 & a_n \\ a_n & a_0 \end{vmatrix}$$

أول عمود وآخر عمود

$$b_1 = \begin{vmatrix} a_0 & a_{n-1} \\ a_n & a_1 \end{vmatrix} \quad ; \quad b_2 = \begin{vmatrix} a_0 & a_{n-2} \\ a_n & a_2 \end{vmatrix}$$

ـــ الشرط الرابع

④ $|b_0| > |b_{n-1}|$

⋮

⑤ $|r_0| > |r_2|$

ⓒ الشرط الأول والثاني لو عوضت بـ $Z = 1$ وطلع الناتج يساوى 0 هتكمل باقى الشروط ولو إتحققت هيبقى (critically stable)

$\underline{Ex}$ $\underline{ch.eqn}$ $1 + \overline{GH}(z) = z^3 - 1.8z^2 + 1.05z - 0.2 = 0$

$F(z) = \underbrace{z^3}_{a_3} - 1.8z^2 + 1.05z \underbrace{-0.2}_{a_0}$

$F(1) > 0$??

$F(1) = 0.05 > 0$ ✓

$(-1)^n F(-1) > 0$??

$-1(-1 - 1.8 - 1.05 - 0.2) = > 0$ ✓

③ $|a_0| < |a_3|$?

0.2 1 $|a_0| < |a_3|$ ✓

	z^0	z^1	z^2	z^3
1	-0.2	1.05	-1.8	1
2	1	-1.8	1.05	-0.2
3	b_0	b_1	b_2	

٣ عناصر (آخر صف)

$$b_0 = \begin{vmatrix} -0.2 & 1 \\ 1 & -0.2 \end{vmatrix} = -0.96$$

$$b_1 = \begin{vmatrix} -0.2 & -1.8 \\ 1 & 1.05 \end{vmatrix} = 1.59$$

$$b_2 = \begin{vmatrix} -0.2 & 1.05 \\ 1 & 1.8 \end{vmatrix} = -0.69$$

④ $|b_0| > |b_2|$?

$0.96 > 0.69$ ✓✓

$\rightsquigarrow$ system is stable.

$\boxed{Ex\,2}$ Ch-equation

$$z^4 - 1.7z^3 + 1.04z^2 - 0.268z + 0.024 = 0$$

① $\underbrace{F(z=1)}_{\to 0.096} > 0$? ✓

② $(-1)^{n=4} F(-1) > 0$? ✓

③ $|a_0| < |a_4|$

$0.024 < 1$ ✓

	z^0	z^1	z^2	z^3	z^4
1	0.024	-0.268	1.04	-1.7	1
2	1	-1.7	1.04	-0.268	0.024
3	b_0	b_1	b_2	b_3	
4	b_3	b_2	b_1	b_0	
5	c_0	c_1	c_2	← آخر صف	

$$b_0 = \begin{vmatrix} 0.024 & 1 \\ 1 & 0.024 \end{vmatrix} \approx -0.999 \simeq -1$$

$$b_1 = \begin{vmatrix} 0.024 & -1.7 \\ 1 & -0.268 \end{vmatrix} = 1.693$$

$$b_2 = \begin{vmatrix} 0.024 & 1.04 \\ 1 & 1.04 \end{vmatrix} = -1.015.$$

$$b_3 = \begin{vmatrix} 0.024 & -0.268 \\ 1 & -1.7 \end{vmatrix} = 0.227$$

(4) $|b_0| = 1 > |b_3| = 0.227$ ✓

$$C_0 = \begin{vmatrix} -1 & 0.227 \\ 0.227 & -1 \end{vmatrix} = 0.943$$

$C_2 = 0.63$

(5) $|C_0| = 0.943 > |C_2| = 0.63$ ✓

$\rightarrow$ System is stable.

Ex :3 ch. eq

$F(z) = z^3 + 3.3 z^2 + 3z + 0.8 = 0$

1) $F(z = 1) = 0$? ✓

2) $(-1)^3 F(-1) > 0$?

$\hookrightarrow = -ve$ (هذا الشرط لم يتحقق)

$\rightarrow$ system unstable.